

Second-Price

Theorem

Truth-telling is a dominant strategy in a second-price auction.

- In fact, we know this already (do you see why?)
- However, we'll look at a simpler, direct proof.

Second-Price proof

Theorem

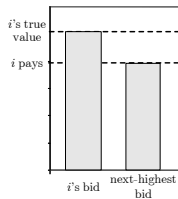
Truth-telling is a dominant strategy in a second-price auction.

Proof.

Assume that the other bidders bid in some arbitrary way. We must show that i 's best response is always to bid truthfully. We'll break the proof into two cases:

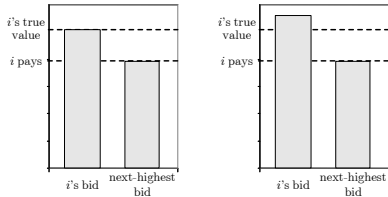
- 1 Bidding honestly, i would win the auction
- 2 Bidding honestly, i would lose the auction

Second-Price proof (2)



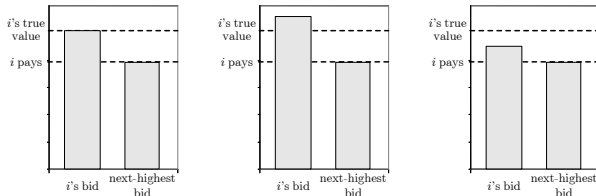
- Bidding honestly, i is the winner

Second-Price proof (2)



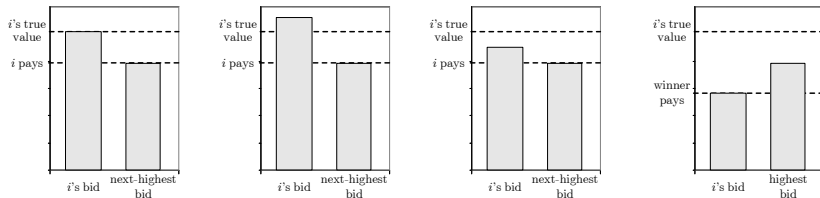
- Bidding honestly, i is the winner
- If i bids higher, he will still win and still pay the same amount

Second-Price proof (2)



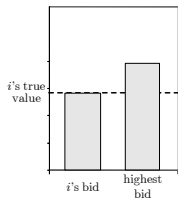
- Bidding honestly, i is the winner
- If i bids higher, he will still win and still pay the same amount
- If i bids lower, he will either still win and still pay the same amount. . .

Second-Price proof (2)



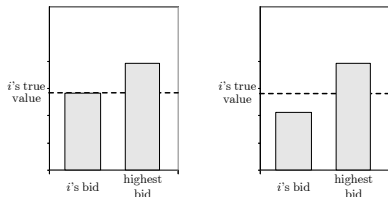
- Bidding honestly, i is the winner
- If i bids higher, he will still win and still pay the same amount
- If i bids lower, he will either still win and still pay the same amount. . . or lose and get utility of zero.

Second-Price proof (3)



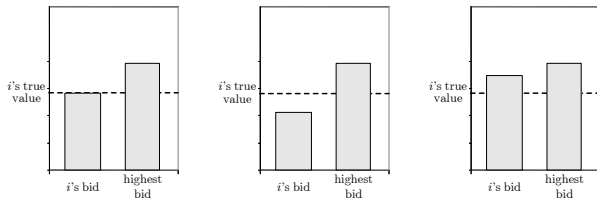
- Bidding honestly, i is not the winner

Second-Price proof (3)



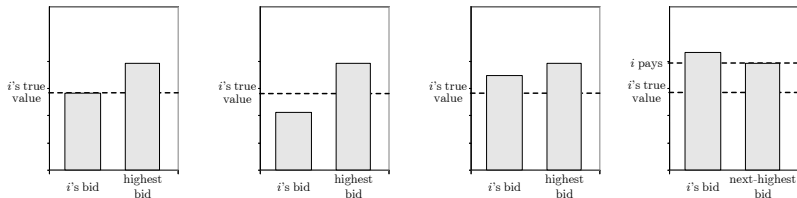
- Bidding honestly, i is not the winner
- If i bids lower, he will still lose and still pay nothing

Second-Price proof (3)



- Bidding honestly, i is not the winner
- If i bids lower, he will still lose and still pay nothing
- If i bids higher, he will either still lose and still pay nothing. . .

Second-Price proof (3)



- Bidding honestly, i is not the winner
- If i bids lower, he will still lose and still pay nothing
- If i bids higher, he will either still lose and still pay nothing. . . or win and pay more than his valuation.

English and Japanese auctions

- A much **more complicated** strategy space
 - extensive form game
 - bidders are able to condition their bids on information revealed by others
 - in the case of English auctions, the ability to place jump bids
- intuitively, though, the revealed information doesn't make any difference in the IPV setting.

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Theorem

*Under the independent private values model (IPV), it is a **dominant strategy** for bidders to bid up to (and not beyond) their valuations in both Japanese and English auctions.*