

# Position Auctions

- Search engines make most of their money—billions of dollars—by selling advertisements through **position auctions**.
  - Keyword-specific “slots” in a list on the right-hand side of a page of search results are simultaneously offered to advertisers.
  - Slots are more valuable the closer they are to the top: more likely to be clicked.
  - Every time a user searches for a keyword, an auction is held.
  - Advertisers pay only if a user clicks on their ad.

# Formal Model

- Define the setting:
  - $N$ : the set of bidders (advertisers)
  - $v_i$ :  $i$ 's (commonly known) valuation for a click
  - $b_i \in \mathbb{R}_+$ :  $i$ 's bid
  - $b_{(j)}$ : the  $j$ th-highest bid, or 0 if there are fewer than  $j$  bids
  - $G = \{1, \dots, m\}$ : the set of goods (slots)
  - $\alpha_j$ : the expected number of clicks (the **click-through rate**) that an ad will receive if it is listed in the  $i$ th slot
- Observe:
  - $\alpha$  does not depend on a bidder's identity
  - the auction is modeled as unrepeated
  - we assume that agents know each other's valuations

# Generalized First-Price Auctions

- The generalized first-price auction was the first position auction to be used by search engines.

## Definition (Generalized first-price auction)

The **generalized first-price auction** (GFP) awards the bidder with the  $j$ th-highest bid the  $j$ th slot. If bidder  $i$ 's ad receives a click, he pays the auctioneer  $b_i$ .

- These auctions do not always have pure-strategy equilibria, even in the unrepeated, perfect-information case.
  - if bidders bid by best responding to each other, their bid amounts can cycle: a low bidder increases bids to try to get a slot; he is outbid by a high bidder; eventually the low bidder drops out; the high bidder reduces his bid; ...

# Generalized Second-Price Auctions

- The instability of bidding under the GFP led to the introduction of the generalized second-price auction.
- It is now the dominant mechanism in practice.

## Definition (Generalized second-price auction)

The **generalized second-price auction** (GSP) awards the bidder with the  $j$ th-highest bid the  $j$ th slot. If bidder  $i$ 's ad is ranked in slot  $j$  and receives a click, he pays the auctioneer  $b_{(j+1)}$ .

# VCG in the position auction setting

- GSP seems very similar to the VCG mechanism. However, these two mechanisms are actually quite different, as becomes clear when we apply the VCG formula to the position auction setting.

## Definition (VCG)

In the position auction setting, the **VCG mechanism** awards the bidder with the  $j$ th-highest bid the  $j$ th slot. If bidder  $i$ 's ad is ranked in slot  $j$  and receives a click, he pays the auctioneer  $\frac{1}{\alpha_j} \sum_{k=j+1}^{m+1} b_{(k)} (\alpha_{k-1} - \alpha_k)$ .

- the key difference: GSP does not charge an agent his social cost, which depends on the differences between click-through rates that other agents would receive with and without his presence.

# Equilibria of GSP

- Truthful bidding is not an equilibrium of the GSP.
- Perfect-information setting: the GSP has many equilibria.
  - the most stable configurations will be **locally envy free**: no bidder will wish that he could switch places with the bidder who won the slot directly above his.
  - There exists a locally envy-free equilibrium of the GSP that achieves exactly the VCG allocations and payments.
  - All other locally envy-free equilibria lead to higher revenues for the seller, and hence are worse for the bidders.
- Beyond perfect information: one can construct a generalized **English** auction that corresponds to the GSP, and to show that this English auction has a unique equilibrium in which payoffs are again the same as the VCG payoffs, and the equilibrium is **ex post**, meaning that it is independent of the underlying valuation distribution.