Valuations for heterogeneous goods

- now consider a case where multiple, heterogeneous goods are being sold.
- consider the sorts of valuations that agents could have in this case:
  - complementarity: for sets S and T, $v(S \cup T) > v(S) + v(T)$
    - e.g., a left shoe and a right shoe
  - substitutability: $v(S \cup T) < v(S) + v(T)$
    - e.g., two tickets to different movies playing at the same time
- substitutability is relatively easy to deal with
  - e.g., just sell the goods sequentially, or allow bid withdrawal
- complementarity is trickier...

Fun Game

1	2	3
4	5	6
7	8	9

- 9 plots of land for sale, geographically related as shown
- IPV, normally distributed with mean 50, stdev 5
- payoff:
  - if you get one good other than #5: v_i
  - any two goods: $3v_i$
  - any three (or more) goods: $5v_i$
- Rules:
  - auctioneer moves from one good to the next sequentially, holding an English auction for each good.
  - bidding stops on a good: move on to the next good
  - no bids for any of the 9 goods: end the auction

Combinatorial auctions

- running a simultaneous ascending auction is inefficient
  - exposure problem
  - inefficiency due to fear of exposure
- if we want an efficient outcome, why not just run VCG?
  - unfortunately, it again requires solving an NP-complete problem
  - let there be n goods, m bids, sets C_j of XOR bids
  - weighted set packing problem:

$$\begin{aligned}
\max \sum_{i=1}^{m} x_i p_i & & \\
\text{subject to} \sum_{i \mid g \in S_i} x_i \leq 1 & & \forall g \\
x_i \in \{0,1\} & & \forall i \\
\sum_{k \in C_j} x_k \leq 1 & & \forall j
\end{aligned}$$

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$$

- we don't need the XOR constraints
  - instead, we can introduce "dummy goods" that don't correspond to goods in the auction, but that enforce XOR constraints.
  - amounts to exactly the same thing: the first constraint has the same form as the third

Winner determination problem

How do we deal with the computational complexity of the winner determination problem?

- Require bids to come from a restricted set, guaranteeing that the WDP can be solved in polynomial time
  - problem: these restricted sets are *very* restricted...
- Use heuristic methods to solve the problem
  - this works pretty well in practice, making it possible to solve WDPs with many hundreds of goods and thousands of bids.