

## VCG

## Definition (Clarke tax)

The **Clarke tax** sets the  $h_i$  term in a Groves mechanism as

$$h_i(\hat{v}_{-i}) = \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}_{-i})).$$

## Definition (Vickrey-Clarke-Groves (VCG) mechanism)

The **Vickrey-Clarke-Groves mechanism** is a direct quasilinear mechanism  $(\chi, p)$ , where

$$\chi(\hat{v}) = \arg \max_x \sum_i \hat{v}_i(x)$$

$$p_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}))$$

# VCG discussion

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- You get paid everyone's utility under the allocation that is actually chosen
  - except your own, but you get that directly as utility
- Then you get charged everyone's utility in the world where you don't participate
- Thus you pay your **social cost**

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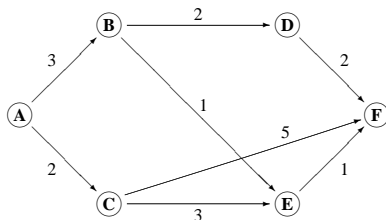


# VCG properties

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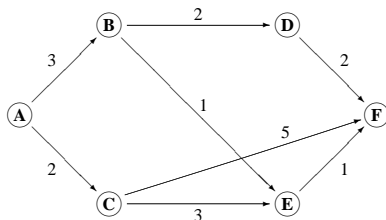
- Because only **pivotal** agents have to pay, VCG is also called the **pivot mechanism**
- It's dominant-strategy truthful, because it's a Groves mechanism

# Selfish routing example



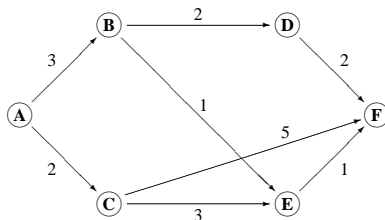
- What outcome will be selected by  $\chi$ ?

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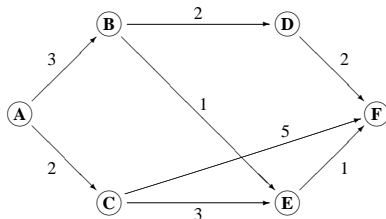
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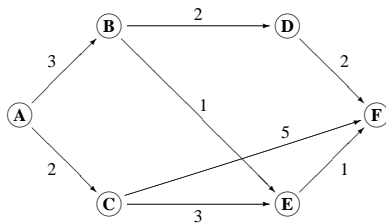
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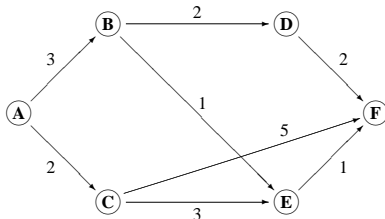
- What outcome will be selected by  $\chi$ ? path  $ABEF$ .
- How much will  $AC$  have to pay?
  - The shortest path taking his declaration into account has a length of 5, and imposes a cost of  $-5$  on agents other than him (because it does not involve him). Likewise, the shortest path without  $AC$ 's declaration also has a length of 5. Thus, his payment  $p_{AC} = (-5) - (-5) = 0$ .
  - This is what we expect, since  $AC$  is not pivotal.
  - Likewise,  $BD$ ,  $CE$ ,  $CF$  and  $DF$  will all pay zero.

# Selfish routing example



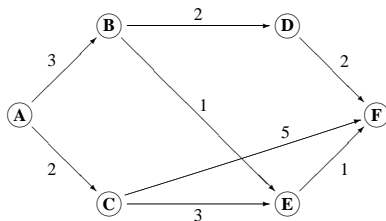
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# Selfish routing example



- How much will  $AB$  pay?
  - The shortest path taking  $AB$ 's declaration into account has a length of 5, and imposes a cost of 2 on other agents.
  - The shortest path without  $AB$  is  $ACEF$ , which has a cost of 6.
  - Thus  $p_{AB} = (-6) - (-2) = -4$ .

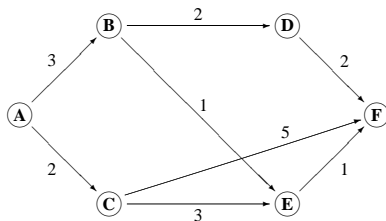
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- How much will  $BE$  pay?

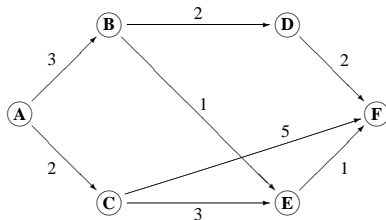


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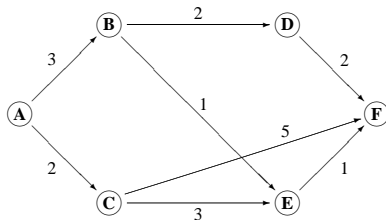
- How much will  $BE$  pay?  $p_{BE} = (-6) - (-4) = -2$ .

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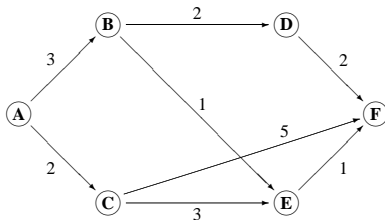
- How much will  $BE$  pay?  $p_{BE} = (-6) - (-4) = -2$ .
- How much will  $EF$  pay?

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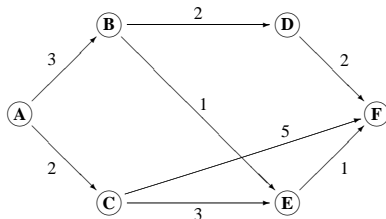
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- How much will  $EF$  pay?  $p_{EF} = (-7) - (-4) = -3$ .

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  - $EF$  and  $BE$  have the same costs but are paid different amounts. Why?

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- How much will  $EF$  pay?  $p_{EF} = (-7) - (-4) = -3$ .
  - $EF$  and  $BE$  have the same costs but are paid different amounts. Why?
  - $EF$  has more *market power*: for the other agents, the situation without  $EF$  is worse than the situation without  $BE$ .