Another property

Definition (No single-agent effect)

An environment exhibits no single-agent effect if $\forall i$, $\forall v_{-i}$, $\forall x \in \arg\max_y \sum_j v_j(y)$ there exists a choice x' that is feasible without i and that has $\sum_{j \neq i} v_j(x') \geq \sum_{j \neq i} v_j(x)$.

Example

Consider a single-sided auction. Dropping an agent just reduces the amount of competition, making the others better off.

Good news

Theorem

The VCG mechanism is weakly budget-balanced when the no single-agent effect property holds.

Proof.

Assume truth-telling in equilibrium. We must show that the sum of transfers from agents to the center is greater than or equal to zero.

$$\sum_i p_i(v) = \sum_i \left(\sum_{j \neq i} v_j(\chi(v_{-i})) - \sum_{j \neq i} v_j(\chi(v)) \right)$$

From the no single-agent effect condition we have that

$$\forall i \; \sum_{j \neq i} v_j(\chi(v_{-i})) \geq \sum_{j \neq i} v_j(\chi(v)).$$

Thus the result follows directly.

More good news

Theorem (Krishna & Perry, 1998)

In any Bayesian game setting in which VCG is ex post individually rational, VCG collects at least as much revenue as any other efficient and ex interim individually-rational mechanism.

- This is somewhat surprising: does not require dominant strategies, and hence compares VCG to all Bayes–Nash mechanisms.
- A useful corollary: VCG is as budget balanced as any efficient mechanism can be
  - it satisfies weak budget balance in every case where *any* dominant strategy, efficient and *ex interim* IR mechanism would be able to do so.

Bad news

Theorem (Green–Laffont; Hurwicz)

No dominant-strategy incentive-compatible mechanism is always both efficient and weakly budget balanced, even if agents are restricted to the simple exchange setting.

Theorem (Myerson–Satterthwaite)

No Bayes-Nash incentive-compatible mechanism is always simultaneously efficient, weakly budget balanced and ex-interim individual rational, even if agents are restricted to quasilinear utility functions.