Besides VCG, what DS mechanisms can we build?

Let $\mathcal{X}_i(\widehat{v}_{-i}) \subseteq X$ denote the set of choices that can be selected by the choice rule χ given the declaration $\widehat{v}_{-i}$ by the agents other than i (i.e., the range of $\chi(\cdot, \widehat{v}_{-i})$).

Theorem

A deterministic mechanism is dominant-strategy truthful if and only if, for every $i \in N$ and every $\widehat{v}_{-i} \in V_{-i}$:

1. *The payment function $p_i(\widehat{v})$ can be written as $p_i(\widehat{v}_{-i}, \chi(\widehat{v}))$;*
2. $\forall v_i \in V_i$, $\chi(v_i, \widehat{v}_{-i}) \in \arg\max_{x \in \mathcal{X}_i(\widehat{v}_{-i})}(v_i(x) - p(\widehat{v}_{-i}, x))$.

- an agent's payment can only depend on other agents' declarations and the selected choice
  - it *cannot* depend otherwise on the agent's own declaration.
- taking the other agent's declarations and the payment function into account, from every player's point of view the mechanism selects the most preferable choice.

Which *Choice Rules* can be Implemented?

Definition (Affine maximizer)

A social choice function is an affine maximizer if it has the form

$$\arg\max_{x \in X} \Big(\gamma_x + \sum_{i \in N} w_i v_i(x) \Big),$$

where each γ_x is an arbitrary constant (may be $-\infty$) and each $w_i \in \mathbb{R}_+$.

Theorem (Roberts)

If there are at least three choices that a social choice function will choose given some input, and if agents have general quasilinear preferences, then the set of (deterministic) social choice functions implementable in dominant strategies is precisely the set of affine maximizers.

Understanding Roberts

- In the case of general quasilinear preferences (i.e., when each agent can have any valuation for each choice $x \in X$) and where the choice function selects from more than two alternatives, affine maximizers are the only DS-implementable social choice functions.

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- Efficiency is an affine-maximizing social choice function for which $\forall x \in X, \gamma_x = 0$ and $\forall i \in N, w_i = 1$.
  - Affine maximizing mechanisms are weighted Groves mechanisms
    - They transform both the choices and the agents' valuations by applying linear weights, then effectively run a Groves mechanism in the transformed space
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  - Thus, we cannot stray very far from Groves mechanisms even if we give up on efficiency
- It is possible to implement a richer set of functions when agents' preferences are restricted further