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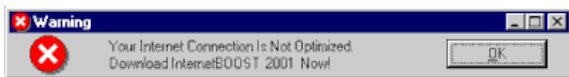
Non-Cooperative Game Theory

- What is it?
 - mathematical study of interaction between **rational**, **self-interested** agents
- Why is it called non-cooperative?
 - while it's most interested in situations where agents' interests conflict, it's not restricted to these settings
 - the key is that the individual is the basic modeling unit, and that individuals pursue their own interests
 - cooperative/coalitional game theory has teams as the central unit, rather than agents

TCP Backoff Game



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Should you send your packets using correctly-implemented TCP (which has a “backoff” mechanism) or using a defective implementation (which doesn’t)?

- Consider this situation as a two-player game:
 - **both use a correct implementation:** both get 1 ms delay
 - **one correct, one defective:** 4 ms delay for correct, 0 ms for defective
 - **both defective:** both get a 3 ms delay.

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- Play this game with someone near you. Then find a new partner and play again. Play five times in total.

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 - **both use a correct implementation:** both get 1 ms delay
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- Questions:
 - What **action** should a player of the game take?
 - Would all users behave **the same** in this scenario?
 - What global **patterns of behaviour** should the system designer expect?
 - Under what **changes to the delay numbers** would behavior be the same?
 - What effect would **communication** have?
 - **Repetitions?** (finite? infinite?)
 - Does it matter if I believe that my opponent is **rational**?

Defining Games

- Finite, n -person game: $\langle N, A, u \rangle$:
 - N is a finite set of n **players**, indexed by i
 - $A = A_1 \times \dots \times A_n$, where A_i is the **action set** for player i
 - $a \in A$ is an **action profile**, and so A is the space of action profiles
 - $u = \langle u_1, \dots, u_n \rangle$, a **utility function** for each player, where $u_i : A \mapsto \mathbb{R}$
- Writing a 2-player game as a **matrix**:
 - row player is player 1, column player is player 2
 - rows are actions $a \in A_1$, columns are $a' \in A_2$
 - cells are outcomes, written as a tuple of utility values for each player

Games in Matrix Form

Here's the **TCP Backoff Game** written as a matrix ("normal form").

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

More General Form

Prisoner's dilemma is any game

	C	D
C	a, a	b, c
D	c, b	d, d

with $c > a > d > b$.

Games of Pure Competition

Players have **exactly opposed** interests

- There must be precisely two players (otherwise they can't have exactly opposed interests)
- For all action profiles $a \in A$, $u_1(a) + u_2(a) = c$ for some constant c
 - Special case: zero sum
- Thus, we only need to store a utility function for one player
 - in a sense, it's a one-player game

Matching Pennies

One player wants to **match**; the other wants to **mismatch**.

	Heads	Tails
Heads	1	-1
Tails	-1	1

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Play this game with someone near you, repeating five times.

Rock-Paper-Scissors

Generalized matching pennies.

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

...Believe it or not, there's an annual international competition for this game!

Games of Cooperation

Players have **exactly the same** interests.

- no conflict: all players want the same things
- $\forall a \in A, \forall i, j, u_i(a) = u_j(a)$
- we often write such games with a single payoff per cell
- why are such games “noncooperative”?

Coordination Game

Which **side of the road** should you drive on?

	Left	Right
Left	1	0
Right	0	1

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	Left	Right
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General Games: Battle of the Sexes

The most interesting games combine elements of *cooperation and competition*.

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

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