Computing Mixed Nash Equilibria: Battle of the Sexes

	B	F
B	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

- It's hard in general to compute Nash equilibria, but it's easy when you can guess the support
- For BoS, let's look for an equilibrium where all actions are part of the support

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- Let player 2 play B with p, F with $1-p$.
- If player 1 best-responds with a mixed strategy, player 2 must make him indifferent between F and B (why?)

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$$
\begin{aligned}
u_1(B) &= u_1(F) \\
2p + 0(1-p) &= 0p + 1(1-p) \\
p &= \frac{1}{3}
\end{aligned}
$$

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  - Why is player 1 willing to randomize?

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- Likewise, player 1 must randomize to make player 2 indifferent.
    - Why is player 1 willing to randomize?
- Let player 1 play B with q, F with $1 - q$.

$$u_2(B) = u_2(F)$$
$$q + 0(1-q) = 0q + 2(1-q)$$
$$q = \frac{2}{3}$$

- Thus the strategies $(\frac{2}{3}, \frac{1}{3})$, $(\frac{1}{3}, \frac{2}{3})$ are a Nash equilibrium.

Interpreting Mixed Strategy Equilibria

What does it mean to play a mixed strategy?

- Randomize to confuse your opponent
  - consider the matching pennies example
- Players randomize when they are uncertain about the other's action
  - consider battle of the sexes
- Mixed strategies are a concise description of what might happen in repeated play: count of pure strategies in the limit
- Mixed strategies describe population dynamics: 2 agents chosen from a population, all having deterministic strategies. MS is the probability of getting an agent who will play one PS or another.

Fun Game!

	L	R
T	$80, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.

Fun Game!

	L	R
T	$320, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.

Fun Game!

	L	R
T	$44, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.

Fun Game!

	L	R
T	$80, 40$; $320, 40$; $44, 40$	$40, 80$
B	$40, 80$	$80, 40$

- Play once as each player, recording the strategy you follow.
- What does row player do in equilibrium of this game?

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    - row player randomizes 50-50 all the time
    - that's what it takes to make column player indifferent

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- What happens when people play this game?

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- Play once as each player, recording the strategy you follow.
- What does row player do in equilibrium of this game?
  - row player randomizes 50-50 all the time
  - that's what it takes to make column player indifferent
- What happens when people play this game?
  - with payoff of 320, row player goes up essentially all the time
  - with payoff of 44, row player goes down essentially all the time