

Maxmin Strategies

- Player i 's **maxmin strategy** is a strategy that maximizes i 's worst-case payoff, in the situation where all the other players (whom we denote $-i$) happen to play the strategies which cause the greatest harm to i .
- The **maxmin value** (or **safety level**) of the game for player i is that minimum amount of payoff guaranteed by a maxmin strategy.

Definition (Maxmin)

The **maxmin strategy** for player i is $\arg \max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$, and the **maxmin value** for player i is $\max_{s_i} \min_{s_{-i}} u_i(s_1, s_2)$.

- Why would i want to play a maxmin strategy?

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- Why would i want to play a maxmin strategy?
 - a conservative agent maximizing worst-case payoff
 - a paranoid agent who believes everyone is out to get him

Minmax Strategies

- Player i 's **minmax strategy** against player $-i$ in a 2-player game is a strategy that minimizes $-i$'s best-case payoff, and the **minmax value** for i against $-i$ is payoff.
- Why would i want to play a minmax strategy?

Definition (Minmax, 2-player)

In a two-player game, the **minmax strategy** for player i against player $-i$ is $\arg \min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$, and player $-i$'s **minmax value** is $\min_{s_i} \max_{s_{-i}} u_{-i}(s_i, s_{-i})$.

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We can generalize to n players.

Definition (Minmax, n -player)

In an n -player game, the **minmax strategy** for player i against player $j \neq i$ is i 's component of the mixed strategy profile s_{-j} in the expression $\arg \min_{s_{-j}} \max_{s_j} u_j(s_j, s_{-j})$, where $-j$ denotes the set of players other than j . As before, the **minmax value** for player j is $\min_{s_{-j}} \max_{s_j} u_j(s_j, s_{-j})$.

Minmax Theorem

Theorem (Minimax theorem (von Neumann, 1928))

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- 1 Each player's maxmin value is equal to his minmax value. By convention, the maxmin value for player 1 is called the **value of the game**.
- 2 For both players, the set of maxmin strategies coincides with the set of minmax strategies.
- 3 Any maxmin strategy profile (or, equivalently, minmax strategy profile) is a Nash equilibrium. Furthermore, these are all the Nash equilibria. Consequently, all Nash equilibria have the same payoff vector (namely, those in which player 1 gets the value of the game).