

Domination

- Let s_i and s'_i be two strategies for player i , and let S_{-i} be the set of all possible strategy profiles for the other players

Definition

s_i **strictly dominates** s'_i if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$

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s_i **weakly dominates** s'_i if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$ and $\exists s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$

Definition

s_i **very weakly dominates** s'_i if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$

Equilibria and dominance

- If one strategy dominates all others, we say it is **dominant**.
- A strategy profile consisting of dominant strategies for every player must be a Nash equilibrium.
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Equilibria and dominance

- If one strategy dominates all others, we say it is **dominant**.
- A strategy profile consisting of dominant strategies for every player must be a Nash equilibrium.
 - An equilibrium in strictly dominant strategies must be unique.
- Consider Prisoner's Dilemma again
 - not only is the only equilibrium the only non-Pareto-optimal outcome, but it's also an equilibrium in strictly dominant strategies!

Traveler's Dilemma

Two travelers purchase identical African masks while on a tropical vacation. Their luggage is lost on the return trip, and the airline asks them to make independent claims for compensation. In anticipation of excessive claims, the airline representative announces: "We know that the bags have identical contents, and we will entertain any claim between \$180 and \$300, but you will each be reimbursed at an amount that equals the minimum of the two claims submitted. If the two claims differ, we will also pay a reward R to the person making the smaller claim and we will deduct a penalty R from the reimbursement to the person making the larger claim."

Traveler's Dilemma

- Action: choose an integer between 180 and 300
- If both players pick the same number, they both get that amount as payoff
- If players pick a different number:
 - the low player gets his number (L) plus some constant R
 - the high player gets $L - R$, $R = 5$.
- Play this game *once* with a partner; play with as many different partners as you like.

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- Play this game *once* with a partner; play with as many different partners as you like.
 - Now set $R = 180$, and again play with as many partners as you like.

Traveler's Dilemma

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- What happens?
 - with $R = 5$ most people choose 295–300
 - with $R = 180$ most people choose 180

Dominated strategies

- No equilibrium can involve a strictly dominated strategy
 - Thus we can remove it, and end up with a strategically equivalent game
 - This might allow us to remove another strategy that wasn't dominated before
 - Running this process to termination is called **iterated removal of dominated strategies**.

Iterated Removal of Dominated Strategies: Example

	L	C	R
U	3, 1	0, 1	0, 0
M	1, 1	1, 1	5, 0
D	0, 1	4, 1	0, 0

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	L	C	R
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- R is dominated by L .

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- M is dominated by the mixed strategy that selects U and D with equal probability.

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- No other strategies are dominated.

Iterated Removal of Dominated Strategies

- This process **preserves Nash equilibria**.
 - strict dominance: all equilibria preserved.
 - weak or very weak dominance: at least one equilibrium preserved.
- Thus, it can be used as a **preprocessing step** before computing an equilibrium
 - Some games are solvable using this technique.
 - Example: Traveler's Dilemma!
- What about the **order of removal** when there are multiple dominated strategies?
 - strict dominance: doesn't matter.
 - weak or very weak dominance: can affect which equilibria are preserved.