Pithy Quote

If there is intelligent life on other planets, in a majority of them, they would have discovered correlated equilibrium before Nash equilibrium.

– Roger Myerson

Examples

- Consider again Battle of the Sexes.
  - Intuitively, the best outcome seems a 50-50 split between (F, F) and (B, B).
  - But there's no way to achieve this, so either someone loses out (unfair) or both players often miscoordinate
- Another classic example: traffic game

	go	$wait$
go	$-100, -100$	$10, 0$
B	$0, 10$	$-10, -10$

Intuition

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  - A traffic light: a fair randomizing device that tells one of the agents to go and the other to wait.
- Benefits:
  - the negative payoff outcomes are completely avoided
  - fairness is achieved
  - the sum of social welfare exceeds that of any Nash equilibrium
- We could use the same idea to achieve the fair outcome in battle of the sexes.
- Our example presumed that everyone perfectly observes the random event; not required.
- More generally, some random variable with a commonly known distribution, and a private signal to each player about the outcome.
  - signal doesn't determine the outcome or others' signals; however, correlated

Formal definition

Definition (Correlated equilibrium)

Given an n-agent game $G = (N, A, u)$, a correlated equilibrium is a tuple (v, π, σ), where v is a tuple of random variables $v = (v_1, \ldots, v_n)$ with respective domains $D = (D_1, \ldots, D_n)$, π is a joint distribution over v, $\sigma = (\sigma_1, \ldots, \sigma_n)$ is a vector of mappings $\sigma_i : D_i \mapsto A_i$, and for each agent i and every mapping $\sigma'_i : D_i \mapsto A_i$ it is the case that

$$\sum_{d \in D} \pi(d) u_i \left(\sigma_1(d_1), \ldots, \sigma_i(d_i), \ldots, \sigma_n(d_n)\right)$$
$$\geq \sum_{d \in D} \pi(d) u_i \left(\sigma_1(d_1), \ldots, \sigma'_i(d_i), \ldots, \sigma_n(d_n)\right).$$

Existence

Theorem

For every Nash equilibrium σ^* there exists a corresponding correlated equilibrium σ.

- This is easy to show:
  - let $D_i = A_i$
  - let $\pi(d) = \prod_{i \in N} \sigma_i^*(d_i)$
  - σ_i maps each d_i to the corresponding a_i.
- Thus, correlated equilibria always exist

Remarks

- Not every correlated equilibrium is equivalent to a Nash equilibrium
  - thus, correlated equilibrium is a weaker notion than Nash
- Any convex combination of the payoffs achievable under correlated equilibria is itself realizable under a correlated equilibrium
  - start with the Nash equilibria (each of which is a CE)
  - introduce a second randomizing device that selects which CE the agents will play
  - regardless of the probabilities, no agent has incentive to deviate
  - the probabilities can be adjusted to achieve any convex combination of the equilibrium payoffs
  - the randomizing devices can be combined