

Introduction

Our setting now:

- a set of outcomes
- agents have preferences across them
- for the moment, we won't consider incentive issues:
 - center knows agents' preferences, or they declare truthfully
- the goal: a social choice function: a mapping from everyone's preferences to a particular outcome, which is enforced
 - how to pick such functions with desirable properties?

Formal model

Definition (Social choice function)

Assume a set of agents $N = \{1, 2, \dots, n\}$, and a set of outcomes (or alternatives, or candidates) O . Let L_- be the set of non-strict total orders on O . A **social choice function** (over N and O) is a function $C : L_-^n \mapsto O$.

Definition (Social welfare function)

Let N, O, L_- be as above. A **social choice function** (over N and O) is a function $C : L_-^n \mapsto L_-$.

Non-Ranking Voting Schemes

- **Plurality**
 - pick the outcome which is preferred by the most people
- **Cumulative voting**
 - distribute e.g., 5 votes each
 - possible to vote for the same outcome multiple times
- **Approval voting**
 - accept as many outcomes as you “like”

Ranking Voting Schemes

- **Plurality with elimination** (“instant runoff”)
 - everyone selects their favorite outcome
 - the outcome with the fewest votes is eliminated
 - repeat until one outcome remains
- **Borda**
 - assign each outcome a number.
 - The most preferred outcome gets a score of $n - 1$, the next most preferred gets $n - 2$, down to the n^{th} outcome which gets 0.
 - Then sum the numbers for each outcome, and choose the one that has the highest score
- **Pairwise elimination**
 - in advance, decide a schedule for the order in which pairs will be compared.
 - given two outcomes, have everyone determine the one that they prefer
 - eliminate the outcome that was not preferred, and continue with the schedule

Condorcet Condition

- If there is a candidate who is preferred to every other candidate in pairwise runoffs, that candidate should be the winner
- While the Condorcet condition is considered an important property for a voting system to satisfy, there is not always a Condorcet winner
- sometimes, there's a cycle where A defeats B , B defeats C , and C defeats A in their pairwise runoffs

Fun Game

- Imagine that there was an opportunity to take a one-week class trip at the end of term, to one of the following destinations:
 - (O) Orlando, FL
 - (P) Paris, France
 - (T) Tehran, Iran
 - (B) Beijing, China
- Construct your preference ordering

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 - plurality (raise hands)

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 - plurality (raise hands)
 - plurality with elimination (raise hands)
 - Borda (volunteer to tabulate)
 - pairwise elimination (raise hands, I'll pick a schedule)

Condorcet example

499 agents: $A \succ B \succ C$

3 agents: $B \succ C \succ A$

498 agents: $C \succ B \succ A$

- What is the Condorcet winner?

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- What would win under plurality with elimination?

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Sensitivity to Losing Candidate

35 agents: $A \succ C \succ B$

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- Now consider dropping C . Now what happens under both Borda and plurality?

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Sensitivity to Agenda Setter

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- Who wins with the ordering B, C, A ? A

Another Pairwise Elimination Problem

1 agent: $B \succ D \succ C \succ A$

1 agent: $A \succ B \succ D \succ C$

1 agent: $C \succ A \succ B \succ D$

- Who wins under pairwise elimination with the ordering A, B, C, D ?

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- Who wins under pairwise elimination with the ordering A, B, C, D ? D .

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- What is the problem with this?

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- Who wins under pairwise elimination with the ordering A, B, C, D ? D .
- What is the problem with this?
 - *all* of the agents prefer B to D —the selected candidate is Pareto-dominated!